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Gauge and gravitational anomalies in $\mathrm{D}=4$ $\mathrm{N}=1$ orientifolds

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Abstract: The cancellation of $U(1)$-gauge and $U(1)$-gravitational anomalies in certain $D=4$ $N=1$ Type IIB orientifolds is analyzed in detail, from a string theory point of view. We verify the proposal that these anomalies are cancelled by a Green-Schwarz mechanism involving only twisted Ramond-Ramond fields. By factorizing one-loop partition functions, we also get the anomalous couplings of D-branes, O-planes and orbifold fixed-points to these twisted fields. Twisted sectors with fixed-planes participate to the inflow mechanism in a peculiar way.

Keywords: D-branes, orientifolds, anomalies.

Contents

1. Introduction

Thanks to the discovery of D-branes and the understanding of their prominent role in string theory [1], new interesting possibilities have opened up to construct phenomenologically viable string vacua. Among others, four dimensional $N = 1$ Type IIB orientifolds [2, 3, 4, 5, 6, 7] represent an example of new perturbative models that have become accessible in alternative to the well-studied heterotic compactifications. An interesting issue in these new vacua is anomaly cancellation. It has been proposed and argued in [8] that both $U(1)$-gauge and $U(1)$-gravitational anomalies are cancelled by a generalization of the $D = 4$ Green-Schwarz (GS) mechanism [9], involving the exchange of twisted Ramond-Ramond (RR) closed string states only. This proposal has been analyzed in detail at the level of low-energy effective action in [10] (see also [11, 12, 13]). A string theory analysis of some of the anomaly cancelling terms for these orientifolds has been given in [14], which is however limited to special models and for gauge anomalies only. Due to the potential relevance of these string vacua in building realistic models (see for instance [15]), it is of interest to perform a more complete analysis of the cancellation of anomalies in these models at the string level.

In [16], a general method for the study of anomalies in string theory vacua, based on the computation of the topological one-loop partition function in the presence of gauge and gravitational backgrounds, has been described. At lowest order in derivatives (momenta), this calculation yields directly the one-loop anomaly from the charged massless (open string) spectrum and *at the same time* (minus) the tree-level inflow of anomaly mediated by neutral massless (closed string) fields. By analyzing the transverse channel, it is possible to determine which states participate to the GS mechanism. On the other hand, the direct channel analysis allows a precise check of the charged spectrum.

Our results are in agreement with the proposal of [8]: the only fields participating to the inflow of anomaly are combinations of twisted RR axions (with twists different from $\mathbf{Z}_2$). Notice that this is in striking contrast with what happens in six-dimensional $N = 1$ IIB orientifolds, where it was explicitly shown in [16] that all closed RR string states, both twisted and untwisted, participate in general to the anomaly cancellation mechanism. We also derive by factorization of the above one-loop partition functions, the Wess-Zumino (WZ) couplings for D9-branes, D5-branes and orbifold fixed-points to the twisted RR axions. Interestingly, the gravitational couplings for fixed-points can be completely reabsorbed in those of the D-branes present in the models, with the net effect of rescaling by a factor of 3/2 the gravitational part.

For the $N = 1$ sectors, *i.e.* those without planes left fixed by the orbifold action, all the anomalous couplings can then be rewritten in a unified way, as reported in the formulae (4.4) and (4.8). The $N = 2$ sectors, containing fixed-planes, are instead more subtle. In these cases, a non-vanishing inflow of anomaly arises only in the 95 sector and correspondingly it is not possible to fix unambiguously the anomalous couplings by factorization. However, one can conclude that neither the D9-branes nor the D5-branes can couple to the simple and natural symmetric combination of the corresponding RR twisted axions, as happens in $N = 1$ sectors. Although we do not have a satisfactory and precise explanation of this fact, we will see that the general form of these couplings might allow interesting tree-level corrections to the gauge couplings, even for unbroken non-Abelian gauge group. This is in contrast to the situation in the $N = 1$ sectors, where the presence of Fayet-Iliopoulos terms, related by supersymmetry to some of the WZ terms above, fixes to zero the tree-level gauge couplings dependence on the twisted Neveu-Schwarz-Neveu-Schwarz (NSNS) scalars, supersymmetric partners of the RR axions. Without entering into all the details of the low-energy effective action, which has been extensively analyzed in [8, 14, 10], we also discuss the spontaneous breaking of $U(1)$ factors through a Higgs mechanism induced by these anomaly-cancelling couplings, both in the $N = 1$ and $N = 2$ sectors.

The paper is organized as follows. In section two, we briefly review some properties of the orientifold models under analysis. In section three, we compute the one-loop partition function in the odd spin-structure yielding the inflow of anomaly. In section four, we deduce then the WZ couplings by factorization. The last section contains a brief field theory analysis of our results. Finally, we report in the appendix the explicit combinations of $U(1)$ gauge fields which become massive.

2. $D = 4$ $N = 1$ orientifolds

In this section, we review some generalities about the $D = 4$ $N = 1$ Type IIB orientifolds we consider. These models always contain 32 D9-branes, required to cancel the tadpoles from the O9-plane associated to the world-sheet parity operator Ω, and 32 D5-brane wrapped along the third compact plane when N is even, required to cancel the tadpoles from the 16 O5-planes associated to the element ΩR of the orientifold group, R being a reflection along the first two compact planes. In the following, we shall restrict to the maximally symmetric case in which all the D5-branes sit at the origin of the first two compact planes.

$\mathbf{Z}_N$ orientifolds

The $\mathbf{Z}_N$ action is generated by the element $\theta = exp(2\pi i v_i J_i)$, where J_i is the rotation generator in the i-th compact plane and v_i are the corresponding components of the twist vector defining the action, $v = (v_1, v_2, v_3)$. In the open string sector, the twist θ^k is represented by matrices γ_k on the Chan-Paton wave function. In a suitable basis, one can choose $\gamma_{k,9} = \gamma_{k,5} = (\gamma)^k$. The $\mathbf{Z}_N$ actions leading to consistent models with cancelled tadpoles are given in the table below.

G	v	γ
$\mathbf{Z}_3$	$(1,1,-2)/3$	$\mathrm{diag}\left(\alpha^2 \mathbf{I}^a_{\mathbf{12}}, \alpha^{-2}\mathbf{I}^a_{\overline{\mathbf{12}}}, \mathbf{I}^b_{\mathbf{8}}\right)$
$\mathbf{Z}_7$	$(1,2,-3)/7$	$\mathrm{diag}\left(\alpha^2 \mathbf{I}^a_{\mathbf{4}}, \alpha^{-2}\mathbf{I}^a_{\overline{\mathbf{4}}}, \alpha^4 \mathbf{I}^b_{\mathbf{4}}, \alpha^{-4}\mathbf{I}^b_{\overline{\mathbf{4}}}, \alpha^6 \mathbf{I}^c_{\mathbf{4}}, \alpha^{-6}\mathbf{I}^c_{\overline{\mathbf{4}}}, \mathbf{I}^d_{\mathbf{8}}\right)$
$\mathbf{Z}_6$	$(1,1,-2)/6$	$\mathrm{diag}\left(\alpha\, \mathbf{I}^a_{\mathbf{6}}, \alpha^{-1}\mathbf{I}^a_{\overline{\mathbf{6}}}, \alpha^5 \mathbf{I}^b_{\mathbf{6}}, \alpha^{-5}\mathbf{I}^b_{\overline{\mathbf{6}}}, \alpha^3 \mathbf{I}^c_{\mathbf{4}}, \alpha^{-3}\mathbf{I}^c_{\overline{\mathbf{4}}}\right)$
$\mathbf{Z}'_6$	$(1,-3,2)/6$	$\mathrm{diag}\left(\alpha\, \mathbf{I}^a_{\mathbf{4}}, \alpha^{-1}\mathbf{I}^a_{\overline{\mathbf{4}}}, \alpha^5 \mathbf{I}^b_{\mathbf{4}}, \alpha^{-5}\mathbf{I}^b_{\overline{\mathbf{4}}}, \alpha^3 \mathbf{I}^c_{\mathbf{8}}, \alpha^{-3}\mathbf{I}^c_{\overline{\mathbf{8}}}\right)$
$\mathbf{Z}_{12}$	$(1,-5,4)/12$	$\mathrm{diag}\left(\alpha^{-1} \mathbf{I}^a_{\mathbf{3}}, \alpha\,\mathbf{I}^a_{\overline{\mathbf{3}}}, \alpha^5 \mathbf{I}^b_{\mathbf{3}}, \alpha^{-5}\mathbf{I}^b_{\overline{\mathbf{3}}}, \alpha^{-7} \mathbf{I}^c_{\mathbf{3}}, \alpha^{7}\mathbf{I}^c_{\overline{\mathbf{3}}}, \alpha^{11} \mathbf{I}^d_{\mathbf{3}}, \alpha^{-11}\mathbf{I}^d_{\overline{\mathbf{3}}},\right.$ $\left.\alpha^3 \mathbf{I}^e_{\mathbf{2}}, \alpha^{-3}\mathbf{I}^e_{\overline{\mathbf{2}}}, \alpha^9 \mathbf{I}^f_{\mathbf{2}}, \alpha^{-9}\mathbf{I}^f_{\overline{\mathbf{2}}}\right)$

Table 1: Twist vectors and matrices for the $\mathbf{Z}_N$ models. Latin letters refer to the various factors of the gauge group reported in table 3, $\mathbf{I}_\rho$ indicates the identity in the representation ρ, and $\alpha = e^{\frac{\pi}{N}i}$.

$\mathbf{Z}_N \times \mathbf{Z}_M$ orientifolds

The $\mathbf{Z}_N \times \mathbf{Z}_M$ action is generated by the elements $\theta = exp(2\pi i v_i J_i)$, $\omega = exp(2\pi i w_i J_i)$, associated to the twist vectors $v = (v_1, v_2, v_3)$ and $w = (w_1, w_2, w_3)$. The matrices γ_k and δ_l representing the twists θ^k and ω^l on open strings can be chosen such that $\gamma_{k,9} = (\gamma)^k$, $\gamma_{k,5} = (-\gamma)^k$, $\delta_{l,9} = \delta_{l,5} = (\delta)^l$ [6]. The $\mathbf{Z}_N \times \mathbf{Z}_M$ actions in table 2 lead to consistent model with cancelled tadpoles.

G	v, w	γ, δ
$\mathbf{Z}_3 \times \mathbf{Z}_3$	$(1,-1,0)/3$, $(0,1,-1)/3$	$\mathrm{diag}\left(\alpha^2\mathbf{I}^a_{\mathbf{4}}, \alpha^{-2}\mathbf{I}^a_{\bar{\mathbf{4}}}, \alpha^4\mathbf{I}^b_{\mathbf{4}}, \alpha^{-4}\mathbf{I}^b_{\bar{\mathbf{4}}}, \mathbf{I}^c_{\mathbf{4}}, \mathbf{I}^c_{\bar{\mathbf{4}}}, \mathbf{I}^d_{\mathbf{8}}\right)$, $\mathrm{diag}\left(\mathbf{I}^a_{\mathbf{4}}, \mathbf{I}^a_{\bar{\mathbf{4}}}, \beta^2\mathbf{I}^b_{\mathbf{4}}, \beta^{-2}\mathbf{I}^b_{\bar{\mathbf{4}}}, \beta^4\mathbf{I}^c_{\mathbf{4}}, \beta^{-4}\mathbf{I}^c_{\bar{\mathbf{4}}}, \mathbf{I}^d_{\mathbf{8}}\right)$
$\mathbf{Z}_3 \times \mathbf{Z}_6$	$(1,0,-1)/3$, $(1,-1,0)/6$	$\mathrm{diag}\left(\alpha^4\mathbf{I}^a_{\mathbf{2}}, \alpha^{-4}\mathbf{I}^a_{\bar{\mathbf{2}}}, \mathbf{I}^b_{\mathbf{2}}, \mathbf{I}^b_{\bar{\mathbf{2}}}, \alpha^2\mathbf{I}^c_{\mathbf{2}}, \alpha^{-2}\mathbf{I}^c_{\bar{\mathbf{2}}}, \mathbf{I}^d_{\mathbf{2}}, \mathbf{I}^d_{\bar{\mathbf{2}}}, \alpha^2\mathbf{I}^e_{\mathbf{2}}, \alpha^{-2}\mathbf{I}^e_{\bar{\mathbf{2}}}, \alpha^4\mathbf{I}^f_{\mathbf{2}}, \alpha^{-4}\mathbf{I}^f_{\bar{\mathbf{2}}}, \mathbf{I}^g_{\mathbf{4}}, \mathbf{I}^g_{\bar{\mathbf{4}}}\right)$, $\mathrm{diag}\left(\beta^{-1}\mathbf{I}^a_{\mathbf{2}}, \beta\,\mathbf{I}^a_{\bar{\mathbf{2}}}, \beta^{-1}\mathbf{I}^b_{\mathbf{2}}, \beta\,\mathbf{I}^b_{\bar{\mathbf{2}}}, \beta^{-5}\mathbf{I}^c_{\mathbf{2}}, \beta^5\mathbf{I}^c_{\bar{\mathbf{2}}}, \beta^{-5}\mathbf{I}^d_{\mathbf{2}}, \beta^5\mathbf{I}^d_{\bar{\mathbf{2}}}, \beta^3\mathbf{I}^e_{\mathbf{2}}, \beta^{-3}\mathbf{I}^e_{\bar{\mathbf{2}}}, \beta^3\mathbf{I}^f_{\mathbf{2}}, \beta^{-3}\mathbf{I}^f_{\bar{\mathbf{2}}}, \beta^3\mathbf{I}^g_{\mathbf{4}}, \beta^{-3}\mathbf{I}^g_{\bar{\mathbf{4}}}\right)$

Table 2: Twist vectors and matrices for the $\mathbf{Z}_N \times \mathbf{Z}_M$ models. Latin letters refer to the various factors of the gauge group reported in table 4, $\mathbf{I}_\rho$ indicates the identity in the representation ρ, and $\alpha = e^{\frac{\pi}{N}i}$, $\beta = e^{\frac{\pi}{M}i}$.

The spectra of these orientifold models have been analysed in [2, 3, 4, 5, 6], but there are some minor discrepancies about the charged states content, aside those arising from some arbitrariness in the choice of the twist matrices in tables 1 and 2. We fix these spectra by comparing the standard field-theory computation and our string theory computation of the anomaly. The result are reported in the tables of next section.

3. Gauge and gravitational anomalies

The models we are considering have in general non-vanishing gauge/gravitational one-loop anomalies, which are expected to be cancelled by an inflow of anomaly through the GS mechanism. The latter can be deduced only from a direct string theory computation. It has been shown in [16] that the total anomaly vanishes for any orientifold model with cancelled tadpoles, as a consequence of the cancellation of two equal and opposite contributions which can be identified with the total one-loop anomaly and the total tree-level inflow respectively. The central result of [16] is that the anomaly and inflow polynomials are both given by the same one-loop partition function in the RR odd spin-structure, in external gauge and gravitational backgrounds [1]. Using this strategy, one can analyze in detail the pattern of anomaly cancellation and deduce by factorization (up to overall signs and trivial rescaling of the fields) the relevant CP-odd couplings in the low-energy action.

Recall that the contribution to the one-loop anomaly polynomial from a chiral spinor in the representation ρ of the gauge group, in a gauge and gravitational background with curvatures F and R, involves the Chern-character of the gauge bundle and

[1] In order to compute the anomaly polynomial and not the anomaly itself, one has to go in two dimensions higher and omit the bosonic zero modes.

the Roof-genus of the tangent bundle, and is given by the famous formula [2]

$$I^{\rho}_{1/2} = \mathrm{ch}_{\rho}(F)\,\widehat{A}(R)\ .$$

It is convenient to decompose all the representations of the $U(n)$ and $SO(n)$ factors as tensor products of two fundamental representations associated to the end-points of open strings. Correspondingly, the Chern character appearing in the anomaly decompose as products of Chern characters $c(F)$ in the fundamental representation,

$$c(F) = \mathrm{ch}_{\mathbf{n}}(F) = \mathrm{tr}_{\mathbf{n}}[e^{iF/2\pi}]\ .$$

The adjoint representations of $U(n)$ and $SO(n)$ do not contribute, and for the antisymmetric representation of $U(n)$ one gets the following decomposition:

$$\mathrm{ch}_{\frac{\mathbf{n(n-1)}}{\mathbf{2}}}(F) = \frac{1}{2}\left[c(F)^2 - c(2F)\right]\ . \tag{3.1}$$

From a string theory point of view, the polynomial associated to both the anomaly and the inflow is given by the total one-loop partition function in the odd spin-structure. Only the annulus and the Möbius strip amplitudes contribute. The Klein bottle contribution vanish because of the impossibility of having the correct number of fermionic zero modes inserted. This reflects the fact that there are no pure gravitational anomalies in four dimensions and only charged states can contribute to the anomaly. Summing over all the D-brane sectors, one gets therefore

$$I = \sum_{\alpha,\beta=9,5} I_{\mathcal{A}}^{\alpha\beta} + \sum_{\alpha=9,5} I_{\mathcal{M}}^{\alpha}\ . \tag{3.2}$$

We shall see in the following that the annulus partition function produces naturally a contribution proportional to $c(F)^2$, the Möbius strip giving instead $c(2F)$. Comparing with (3.1), this observation allows to reconstruct the anomalous charged matter spectrum of each model from the string theory expression for the anomaly polynomial.

3.1 $\mathbf{Z}_N$ orientifolds

In the operatorial formalism, the relevant odd spin-structure amplitudes to compute on the annulus and the Möbius strip are encoded in the following partition functions:

$$I_{\mathcal{A}} = \frac{1}{4N}\sum_{k=0}^{N-1} \mathrm{Tr}_R\,[\theta^k\,(-1)^F\,e^{-tH(F,R)}]\ , \tag{3.3}$$

$$I_{\mathcal{M}} = \frac{1}{4N}\sum_{k=0}^{N-1} \mathrm{Tr}_R\,[\Omega\,\theta^k\,(-1)^F\,e^{-tH(F,R)}]\ , \tag{3.4}$$

[2]The corresponding anomaly is given by the Wess-Zumino descent $A = 2\pi i \int I^{(1)}$.

where a sum over Chan-Paton indices is understood. Their evaluation is rather straightforward. Being topological indices, only massless modes contribute, massive states cancelling by supersymmetry. It is convenient to define, in each compact plane $i = 1, 2, 3$ and each k-twisted sector, $k = 1, ..., N-1$, the quantities

$$s_k^i = 2\sin\pi k v_i \ , \quad c_k^i = 2\cos\pi k v_i \ .$$

The number of k-fixed-points, that is those points which are fixed under k-twists, is $N_k^i = (s_k^i)^2$ "per plane", and in total one has $N_k = N_k^1 N_k^2 N_k^3$ fixed-points in the whole compact space and N_k^3 in the third plane, where the D5-branes wrap. Also, it is natural to introduce the modified $\mathbf{Z}_N$ Chern-character

$$\mathrm{ch}_k(F) = \mathrm{tr}[\gamma_k \, e^{iF/2\pi}] \ , \tag{3.5}$$

where the trace is in the Chan-Paton representation.

The untwisted $k = 0$ sector vanish for all the surfaces and D-brane sectors, because of the presence of unsoaked fermionic zero modes in the internal directions, and we restrict therefore to $k \neq 0$. On the annulus, a pair of bosons and fermions with Neumann Neumann (NN) boundary conditions in the i-th compact plane contribute $(s_k^i)^{-2}$ and s_k^i respectively, whereas a pair of bosons and fermions with Dirichlet Dirichlet (DD) boundary conditions give 1 and s_k^i. No fields with ND boundary conditions contribute, having a half-integer mode expansion and correspondingly no zero-energy states. The fields in the non-compact directions give instead the contribution $i\,\mathrm{ch}_k(F_\alpha)\,\mathrm{ch}_k(F_\beta)\,\widehat{A}(R)$ in the $\alpha\beta$ sector. Taking into account the number of fixed-points, N_k in the 99 sector and N_k^3 in the 55 and 95 sectors, one finds

$$I_{\mathcal{A}}^{\alpha\beta} = \frac{i}{4N} \sum_{k=1}^{N-1} C_k^{\alpha\beta}\, \mathrm{ch}_k(F_\alpha)\, \mathrm{ch}_k(F_\beta)\, \widehat{A}(R) \ , \tag{3.6}$$

where

$$C_k^{\alpha\beta} = \begin{cases} s_k^1 s_k^2 s_k^3 \ , & \alpha = \beta \\ s_k^3 \ , & \alpha \neq \beta \ . \end{cases} \tag{3.7}$$

On the Möbius strip a pair of N bosons and fermions in the i-th compact plane give the contributions $(s_k^i)^{-2}$ and s_k^i respectively, whereas a pair of D bosons and fermions give respectively 1 and c_k^i. The fields in the non-compact directions give instead the contribution $i\,\mathrm{ch}_{2k}(2F_\alpha)\,\widehat{A}(R)$ in the α sector. Taking again into account the number of fixed-points, N_k in the 9 sector and N_k^3 in the 5 sector, one finds

$$I_{\mathcal{M}}^{\alpha} = -\frac{i}{4N} \sum_{k=1}^{N-1} C_k^{\alpha}\, \mathrm{ch}_{2k}(2F_\alpha)\, \widehat{A}(R) \ , \tag{3.8}$$

where

$$C_k^{\alpha} = \begin{cases} s_k^1 s_k^2 s_k^3 \ , & \alpha = 9 \\ c_k^1 c_k^2 s_k^3 \ , & \alpha = 5 \ . \end{cases} \tag{3.9}$$

One can check case by case that the two trigonometric factors in $I_{\mathcal{M}}^{9,5}$ give always identical contributions, making manifest the $9 \leftrightarrow 5$ symmetry implied by T-duality.

The massless open string spectrum which is in agreement with the one-loop anomaly (3.2) is reported in table 3.

G	Gauge Group	99/55 Matter	95 Matter
$\mathbf{Z}_3$	$U(12)\times SO(8)$	$3(\mathbf{12},\mathbf{8}),\ 3(\overline{\mathbf{66}},\mathbf{1})$	-
$\mathbf{Z}_7$	$U(4)^3\times SO(8)$	$(\underline{\mathbf{4},\mathbf{1},\mathbf{1}},\mathbf{8}),\ (\underline{\mathbf{6},\mathbf{1},\mathbf{1}},\mathbf{1})$ $(\underline{\bar{\mathbf{4}},\bar{\mathbf{4}},\mathbf{1}},\mathbf{1}),\ (\underline{\bar{\mathbf{4}},\mathbf{1},\mathbf{4}},\mathbf{1})$	-
$\mathbf{Z}_6$	$\left(U(6)^2\times U(4)\right)^2$	$2(\mathbf{15},\mathbf{1},\mathbf{1}),\ 2(\mathbf{1},\overline{\mathbf{15}},\mathbf{1})$ $2(\bar{\mathbf{6}},\mathbf{1},\mathbf{4}),\ 2(\mathbf{1},\mathbf{6},\bar{\mathbf{4}})$ $(\bar{\mathbf{6}},\mathbf{1},\bar{\mathbf{4}}),\ (\mathbf{1},\mathbf{6},\mathbf{4})$ $(\mathbf{6},\bar{\mathbf{6}},\mathbf{1})$	$(\mathbf{6},\mathbf{1},\mathbf{1};\mathbf{6},\mathbf{1},\mathbf{1})$ $(\mathbf{1},\bar{\mathbf{6}},\mathbf{1};\mathbf{1},\bar{\mathbf{6}},\mathbf{1})$ $(\mathbf{1},\mathbf{6},\mathbf{1};\mathbf{1},\mathbf{1},\bar{\mathbf{4}})$ $(\mathbf{1},\mathbf{1},\bar{\mathbf{4}};\mathbf{1},\mathbf{6},\mathbf{1})$ $(\bar{\mathbf{6}},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{4})$ $(\mathbf{1},\mathbf{1},\mathbf{4};\bar{\mathbf{6}},\mathbf{1},\mathbf{1})$
$\mathbf{Z}_6'$	$\left(U(4)^2\times U(8)\right)^2$	$(\mathbf{4},\mathbf{1},\mathbf{8}),\ (\bar{\mathbf{4}},\mathbf{1},\mathbf{8})$ $(\mathbf{1},\mathbf{4},\bar{\mathbf{8}}),\ (\mathbf{1},\bar{\mathbf{4}},\bar{\mathbf{8}})$ $(\mathbf{1},\mathbf{1},\mathbf{28}),\ (\mathbf{1},\mathbf{1},\overline{\mathbf{28}})$ $(\mathbf{6},\mathbf{1},\mathbf{1}),\ (\mathbf{1},\bar{\mathbf{6}},\mathbf{1})$ $(\mathbf{4},\mathbf{4},\mathbf{1}),\ (\bar{\mathbf{4}},\bar{\mathbf{4}},\mathbf{1})$ $(\bar{\mathbf{4}},\mathbf{4},\mathbf{1})$	$(\bar{\mathbf{4}},\mathbf{1},\mathbf{1};\bar{\mathbf{4}},\mathbf{1},\mathbf{1})$ $(\mathbf{1},\mathbf{4},\mathbf{1};\mathbf{1},\mathbf{4},\mathbf{1})$ $(\mathbf{1},\bar{\mathbf{4}},\mathbf{1};\mathbf{1},\mathbf{1},\mathbf{8})$ $(\mathbf{1},\mathbf{1},\mathbf{8};\mathbf{1},\bar{\mathbf{4}},\mathbf{1})$ $(\mathbf{4},\mathbf{1},\mathbf{1};\mathbf{1},\mathbf{1},\bar{\mathbf{8}})$ $(\mathbf{1},\mathbf{1},\bar{\mathbf{8}};\mathbf{4},\mathbf{1},\mathbf{1})$
$\mathbf{Z}_{12}$	$\left(U(3)^4\times U(2)^2\right)^2$	$(\underline{\bar{\mathbf{3}},\mathbf{1}},\underline{\bar{\mathbf{3}},\mathbf{1}},\mathbf{1},\mathbf{1})$ $(\underline{\bar{\mathbf{3}}_{\mathbf{A}},\mathbf{1},\mathbf{1},\mathbf{1}},\mathbf{1},\mathbf{1})$ $(\underline{\mathbf{3},\mathbf{1}},\mathbf{1},\mathbf{1},\underline{\mathbf{2},\mathbf{1}})$ $(\mathbf{1},\mathbf{1},\underline{\mathbf{3},\mathbf{1}},\underline{\bar{\mathbf{2}},\mathbf{1}})$ $(\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1},\bar{\mathbf{2}},\mathbf{1})$ $(\mathbf{1},\mathbf{1},\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{2})$ $(\mathbf{3},\mathbf{1},\mathbf{1},\mathbf{1},\mathbf{1},\bar{\mathbf{2}})$	$(\bar{\mathbf{3}},\mathbf{1}^{\mathbf{5}};\mathbf{1},\bar{\mathbf{3}},\mathbf{1}^{\mathbf{4}})$ $(\mathbf{1}^{\mathbf{2}},\bar{\mathbf{3}},\mathbf{1}^{\mathbf{3}};\mathbf{1}^{\mathbf{3}},\bar{\mathbf{3}},\mathbf{1}^{\mathbf{2}})$ $(\mathbf{3},\mathbf{1}^{\mathbf{5}};\mathbf{1}^{\mathbf{4}},\bar{\mathbf{2}},\mathbf{1})$ $(\mathbf{1}^{\mathbf{2}},\mathbf{3},\mathbf{1}^{\mathbf{3}};\mathbf{1}^{\mathbf{4}},\mathbf{2},\mathbf{1})$ $(\mathbf{1},\mathbf{3},\mathbf{1}^{\mathbf{4}};\mathbf{1}^{\mathbf{5}},\bar{\mathbf{2}})$ $(\mathbf{1}^{\mathbf{3}},\mathbf{3},\mathbf{1}^{\mathbf{2}};\mathbf{1}^{\mathbf{5}},\mathbf{2})$ $+\,(9\leftrightarrow 5)$

Table 3: Massless open string spectrum for Z_N models. The underlined bar means that all the cyclic permutations have to be considered. Our conventions for $U(1)$ charges are such that the $\mathbf{n}$ and $\bar{\mathbf{n}}$ of $U(n)$ carry ± 1 charge with respect to the corresponding $U(1)$.

As a consequence of tadpole cancellation, all irreducible non-Abelian gauge anomalies vanish. Therefore, only mixed $U(1)$-gauge and $U(1)$-gravitational anomalies arise. The former gets contribution only from the annulus, whereas for the latter both the annulus and the Möbius strip contribute. The explicit form of the total anomaly is

$$
\begin{aligned}
I = \frac{1}{4N(2\pi)^3} \sum_{k=1}^{N-1} \Big\{ & \sum_{\alpha,\beta=9,5} C_k^{\alpha\beta}\, \mathrm{tr}(\gamma_k F_\alpha)\, \mathrm{tr}(\gamma_k F_\beta^2) \\
& + \frac{1}{3} \Big[\sum_{\alpha,\beta=9,5} C_k^{\alpha\beta}\, \mathrm{tr}(\gamma_{k,\alpha})\, \mathrm{tr}(\gamma_k F_\beta^3) - 4 \sum_{\alpha=9,5} C_k^{\alpha}\, \mathrm{tr}(\gamma_{2k} F_\alpha^3) \Big] \\
& - \frac{1}{24} \Big[\sum_{\alpha,\beta=9,5} C_k^{\alpha\beta}\, \mathrm{tr}(\gamma_{k,\alpha})\, \mathrm{tr}(\gamma_k F_\beta) - \sum_{\alpha=9,5} C_k^{\alpha}\, \mathrm{tr}(\gamma_{2k} F_\alpha) \Big] \mathrm{tr} R^2 \Big\} \; .
\end{aligned} \tag{3.10}
$$

The cancellation of all the non-Abelian gauge anomalies (second raw of (3.10)) requires the non-trivial identity

$$
\sum_{k=1}^{N-1} C_k^{\alpha}\, \mathrm{tr}(\gamma_{2k} F_\alpha^{2n+1}) = \frac{1}{4} \sum_{k=1}^{N-1} \sum_{\beta=9,5} C_k^{\alpha\beta}\, \mathrm{tr}(\gamma_{k,\beta})\, \mathrm{tr}(\gamma_k F_\alpha^{2n+1}) \; . \tag{3.11}
$$

This is indeed implied by the tadpole-cancellation conditions [17]. Using (3.11), the total mixed $U(1)$-gauge/gravitational anomaly is finally found to be given by the simple expression

$$
I = \frac{1}{4N(2\pi)^3} \sum_{k=1}^{N-1} \sum_{\alpha,\beta=9,5} C_k^{\alpha\beta}\, \mathrm{tr}(\gamma_k F_\alpha) \left(\mathrm{tr}(\gamma_k F_\beta^2) - \frac{1}{32}\, \mathrm{tr}(\gamma_{k,\beta})\, \mathrm{tr} R^2 \right) \; . \tag{3.12}
$$

This result is in agreement with equations (3.5) and (3.8) of [8], corresponding to the first and second term respectively.

3.2 $\mathbf{Z}_N \times \mathbf{Z}_M$ orientifolds

The relevant odd spin-structure partition functions to be computed are now given by the following expressions

$$
I_{\mathcal{A}} = \frac{1}{4NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \mathrm{Tr}_R \left[\theta^k\, \omega^l\, (-1)^F\, e^{-tH(F,R)} \right] , \tag{3.13}
$$

$$
I_{\mathcal{M}} = \frac{1}{4NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} \mathrm{Tr}_R \left[\Omega\, \theta^k\, \omega^l\, (-1)^F\, e^{-tH(F,R)} \right] . \tag{3.14}
$$

Their evaluation is again quite straightforward and the total inflow is given by (3.2). As before, it is very convenient to define, in each compact plane $i = 1, 2, 3$ and each (k,l)-twisted sector, $k = 0, ..., N-1$, $l = 0, ..., M-1$, $(k,l) \neq (0,0)$, the quantities

$$
s_{k,l}^i = 2 \sin \pi (k v_i + l w_i) \; , \quad c_{k,l}^i = 2 \cos \pi (k v_i + l w_i) \; .
$$

The number of (k,l)-fixed-points is $N_{k,l}^i = (s_{k,l}^i)^2$ "per plane", and in total there are $N_{k,l} = N_{k,l}^1 N_{k,l}^2 N_{k,l}^3$ fixed-points in the whole compact space and $N_{k,l}^3$ in the third plane, where the D5-branes wrap. Also, the appropriate $\mathbf{Z}_N \times \mathbf{Z}_M$ Chern-character is

$$
\mathrm{ch}_{k,l}(F) = \mathrm{tr}[\gamma_k\, \delta_l\, e^{iF/2\pi}] \; . \tag{3.15}
$$

The evaluation of (3.13) and (3.14) proceeds along the lines of the previous case. For the annulus, one gets

$$I_{\mathcal{A}}^{\alpha\beta} = \frac{i}{4NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} C_{k,l}^{\alpha\beta} \, \mathrm{ch}_{k,l}(F_\alpha) \, \mathrm{ch}_{k,l}(F_\beta) \, \widehat{A}(R) \ , \tag{3.16}$$

where

$$C_{k,l}^{\alpha\beta} = \begin{cases} s_{k,l}^1 s_{k,l}^2 s_{k,l}^3 \ , & \alpha = \beta \\ s_{k,l}^3 \ , & \alpha \neq \beta \end{cases} \tag{3.17}$$

and for the Möbius strip

$$I_{\mathcal{M}}^{\alpha} = -\frac{i}{4NM} \sum_{k=0}^{N-1} \sum_{l=0}^{M-1} C_{k,l}^{\alpha} \, \mathrm{ch}_{2k,2l}(2F_\alpha) \, \widehat{A}(R) \ , \tag{3.18}$$

where

$$C_{k,l}^{\alpha} = \begin{cases} s_{k,l}^1 s_{k,l}^2 s_{k,l}^3 \ , & \alpha = 9 \\ c_{k,l}^1 c_{k,l}^2 s_{k,l}^3 \ , & \alpha = 5 \ . \end{cases} \tag{3.19}$$

Again, one can check that the two trigonometric factors in $Z_{\mathcal{M}}^{9,5}$ give identical contributions.

The massless open string spectrum corresponding to the one-loop anomaly (3.2) is reported in table 4.

G	Gauge Group	99/55 Matter	95 Matter
$\mathbf{Z}_3 \times \mathbf{Z}_3$	$U(4)^3 \times SO(8)$	$(\underline{\mathbf{4},\mathbf{1},\mathbf{1}},\mathbf{8})$, $(\underline{\bar{\mathbf{6}},\mathbf{1},\mathbf{1}},\mathbf{1})$, $(\underline{\bar{\mathbf{4}},\bar{\mathbf{4}},\mathbf{1}},\mathbf{1})$	-
$\mathbf{Z}_3 \times \mathbf{Z}_6$	$\left(U(2)^6 \times U(4)\right)^2$	$(\mathbf{1},\mathbf{1_A},\mathbf{1^5})$, $(\mathbf{1^3},\bar{\mathbf{1}}_\mathbf{A},\mathbf{1^3})$ $(\mathbf{2},\bar{\mathbf{2}},\mathbf{1^5})$, $(\bar{\mathbf{2}},\bar{\mathbf{2}},\mathbf{1^5})$ $(\mathbf{1^2},\bar{\mathbf{2}},\mathbf{2},\mathbf{1^3})$, $(\mathbf{1^2},\mathbf{2},\mathbf{2},\mathbf{1^3})$ $(\mathbf{1^4},\mathbf{2},\bar{\mathbf{2}},\mathbf{1})$ $(\bar{\mathbf{2}},\mathbf{1^3},\bar{\mathbf{2}},\mathbf{1^2})$, $(\mathbf{1},\mathbf{2},\mathbf{1^2},\mathbf{2},\mathbf{1^2})$ $(\mathbf{1^2},\bar{\mathbf{2}},\mathbf{1},\bar{\mathbf{2}},\mathbf{1^2})$, $(\mathbf{2},\mathbf{1^4},\mathbf{2},\mathbf{1})$ $(\mathbf{1^3},\bar{\mathbf{2}},\mathbf{1},\bar{\mathbf{2}},\mathbf{1})$, $(\mathbf{1^2},\mathbf{2},\mathbf{1^2},\mathbf{2},\mathbf{1})$ $(\mathbf{1},\bar{\mathbf{2}},\mathbf{1^4},\bar{\mathbf{4}})$, $(\mathbf{1^3},\mathbf{2},\mathbf{1^2},\mathbf{4})$ $(\mathbf{1^4},\bar{\mathbf{2}},\mathbf{1},\mathbf{4})$, $(\mathbf{1^5},\mathbf{2},\bar{\mathbf{4}})$	$(\mathbf{2},\mathbf{1^6};\mathbf{1},\bar{\mathbf{2}},\mathbf{1^5})$ $(\mathbf{1^2},\bar{\mathbf{2}},\mathbf{1^4};\mathbf{1^3},\mathbf{2},\mathbf{1^3})$ $(\mathbf{1^4},\mathbf{2},\mathbf{1^2};\mathbf{1^5},\bar{\mathbf{2}},\mathbf{1})$ $(\mathbf{1^5},\mathbf{2},\mathbf{1};\mathbf{1^6},\bar{\mathbf{4}})$ $(\mathbf{1^4},\bar{\mathbf{2}},\mathbf{1^2};\mathbf{1^6},\mathbf{4})$ $+\,(9 \leftrightarrow 5)$

Table 4: Massless open string spectrum for $\mathbf{Z}_N \times \mathbf{Z}_M$ models.

As before, all irreducible non-Abelian gauge anomalies vanish due to a cancellation between the annulus and the Möbius strip contributions, and only mixed $U(1)$-gauge and $U(1)$-gravitational anomalies arise. More precisely, the condition imposed by the vanishing of all irreducible non-Abelian gauge anomalies is

$$\sum_{k=0}^{N-1}\sum_{l=0}^{M-1} C^{\alpha}_{k,l}\,\mathrm{tr}(\gamma_{2k}\delta_{2l}F_{\alpha}^{2n+1}) = \frac{1}{4}\sum_{k=0}^{N-1}\sum_{l=0}^{M-1}\sum_{\beta=9,5} C^{\alpha\beta}_{k,l}\,\mathrm{tr}(\gamma_{k,\beta}\delta_{l,\beta})\,\mathrm{tr}(\gamma_k\delta_l F_{\alpha}^{2n+1})\ . \tag{3.20}$$

Using this condition, the total $U(1)$-gauge/gravitational anomaly takes finally the following simple form:

$$I = \frac{1}{4NM(2\pi)^3}\sum_{k=0}^{N-1}\sum_{l=0}^{M-1}\sum_{\alpha,\beta=9,5} C^{\alpha\beta}_{k,l}\,\mathrm{tr}(\gamma_k\delta_l F_{\alpha})\left(\mathrm{tr}(\gamma_k\delta_l F_{\beta}^2) - \frac{1}{32}\mathrm{tr}(\gamma_{k,\beta}\delta_{l,\beta})\,\mathrm{tr}R^2\right). \tag{3.21}$$

4. Factorization

Having computed all the inflows, the anomalous couplings to RR fields can be obtained by factorization, in the spirit of [18] (see also [19]). In order to do so, one needs a precise knowledge of the massless closed string spectra, which have been studied in detail in [5, 10]. Analyzing (3.6)-(3.8) and (3.16)-(3.18) in the transverse channel, it is then possible to identify, in most cases, which of these states mediate the inflows. In four dimensions, the only states involved in the GS mechanism are axionic scalars and their dual 2-forms in the RR sector, belonging to linear multiplets [3].

As anticipated in the introduction, the analysis of the $N = 2$ sectors containing fixed-planes is complicated by the fact that the inflows in the 99 and 55 sectors vanish. Also the $N = 4$ untwisted sector present some particularities, although it yields a trivially vanishing inflow. We will therefore analyze separately the $N = 1$, $N = 2$ and $N = 4$ sectors.

4.1 $N = 1$ sectors

For $N = 1$ sectors, we assume that both D9 and D5-branes couple in a symmetric way to all the twisted states arising at orbifold fixed-points contained in their world-volume [4]. As we will see, it seems that this quite reasonable assumption needs to be relaxed, in order to explain the factorization in $N = 2$ sectors.

$\mathbf{Z}_N$ models

For $\mathbf{Z}_N$ models, the orientifold projection relates the k and $(N-k)$-twisted sectors, which together yield always linear multiplets [10]. The distinct twisted sectors are

[3] Throughout this section, for simplicity we will always write anomalous couplings both for scalars and their dual 2-forms, despite the well-known fact that there is no local and covariant Lagrangian in which a potential and its dual can appear at the same time. More precisely, one should write the couplings to the dual potential as corrections to the kinetic terms for the field strength of the original potential.

[4] Clearly, this assumption is needed in our factorization procedure, but by a direct computation on the disk or crosscap one should be able to find the correct combination of fields.

therefore labeled by $k = 1, ..., [(N-1)/2]$. The $N = 1$ sectors are those with $k \in S$, where

$$S = \Big\{k = 1, ..., [(N-1)/2] \,|\, N_k \neq 0\Big\} \,.$$

To facilitate the analysis, it is convenient to keep all the N_k linear multiplets arising from the k and $(N-k)$-twisted sectors as independent states, although in general only $N'_k \leq N_k$ of them are independent. The physical N'_k propagating states are identified after suitable $\mathbf{Z}_N$-projections, as in [16, 10], and are reported in the appendix.

In order to figure out which states are responsible for the various inflows, recall that the insertion of θ^k acts as a k-twist in (3.3) and as a $2k$-twist in (3.4), for the closed string state exchanged in the transverse channel. Untwisted closed string exchange can arise only from the $k = 0$ part of (3.6) and the $k = 0$ or $k = N/2$ (when present) parts of (3.8). The $k = 0$ part is always trivially vanishing and from the form of v_3 (see table 1) it is easy to see that the same is true for the $k = N/2$ part. Analogously, no $N/2$-twisted closed string states can contribute to the inflow: the relevant contribution is the $k = N/2$ part of (3.6), which vanishes. There is therefore a non-vanishing contribution only from k-twisted closed strings, with $k \neq 0, N/2$. In order to make the corresponding inflow more explicit, it suffice to group the k and $N-k$ term of each sum in (3.6) and (3.8). The relevant 6-form component of the inflow can then be rewritten as a sum over $k \in S$ only, of the following expressions:

$$\begin{aligned}
I_{\mathcal{A}}^{(k)99}(R, F_9) &= \frac{i}{2} N_k \, Z_{(k)}^9(F_9, R) \, Z_{(k)}^9(F_9, R) \,, \\
I_{\mathcal{A}}^{(k)55}(R, F_5) &= \frac{i}{2} \, N_k^3 \, Z_{(k)}^5(F_5, R) \, Z_{(k)}^5(F_5, R) \,, \\
I_{\mathcal{A}}^{(k)95}(R, F_5, F_9) &= i \, N_k^3 \, Z_{(k)}^9(F_9, R) \, Z_{(k)}^5(F_5, R) \,, \\
I_{\mathcal{M}}^{(2k)9}(R, F_9) &= i \, N_k \, Z_{(2k)}^9(F_9, R) \, Z_{(2k)}(R) \,, \\
I_{\mathcal{M}}^{(2k)5}(R, F_5) &= i \, N_k^3 \, Z_{(2k)}^5(F_5, R) \, Z_{(2k)}(R) \,,
\end{aligned} \tag{4.1}$$

where

$$\begin{aligned}
Z_{(k)}^5(F_5, R) &= \frac{\alpha_k^5}{\sqrt{N}} \sqrt{\left|\frac{s_k^1 s_k^2}{s_k^3}\right|} \, \mathrm{ch}(\gamma_k \, \epsilon_k F_5) \, \sqrt{\widehat{A}(R)} \,, \\
Z_{(k)}^9(F_9, R) &= \frac{\alpha_k^9}{\sqrt{N}} \sqrt{\left|\frac{1}{s_k^1 s_k^2 s_k^3}\right|} \, \mathrm{ch}(\gamma_k \, \epsilon_k F_9) \, \sqrt{\widehat{A}(R)} \,, \\
Z_{(2k)}(R) &= -\frac{4 \, \epsilon_k}{\sqrt{N}} \sqrt{\left|\frac{c_k^1 c_k^2 c_k^3}{s_k^1 s_k^2 s_k^3}\right|} \, \sqrt{\widehat{L}(R/4)} \,.
\end{aligned} \tag{4.2}$$

The coefficients ϵ_k and α_k^α are suitable signs, which are required when some of the s_k^i are negative and can be chosen as $\epsilon_k = \mathrm{sign}(s_k^1 s_k^2 s_k^3)$, $\alpha_k^5 = \mathrm{sign}(s_k^3)$, $\alpha_k^9 = \mathrm{sign}(s_k^1 s_k^2 s_k^3)$.

The corresponding anomalous couplings are

$$S_{D5}^{(k)} = \sqrt{2\pi} \sum_{i_k=1}^{N_k^3} C_{(k)}^{i_k} \wedge Z_{(k)}^5 \ ,$$

$$S_{D9}^{(k)} = \sqrt{2\pi} \sum_{i_k=1}^{N_k} C_{(k)}^{i_k} \wedge Z_{(k)}^9 \ ,$$

$$S_{Fk}^{(2k)} = \sqrt{2\pi} \sum_{i_k=1}^{N_k} C_{(2k)}^{i_k} \wedge Z_{(2k)} \ , \tag{4.3}$$

where $C_{(k)}^{i_k} = \chi_{(k)}^{i_k} + \tilde{\chi}_{(k)}^{i_k}$ is the sum of the RR axions $\chi_{(k)}^{i_k}$ and their duals $\tilde{\chi}_{(k)}^{i_k}$ at each fixed-point; it is understood that one has to keep only the appropriate 6-form component of the integrands. The last contribution refers to the k-fixed-points, that indeed do have anomalous couplings to the gravitational curvature, in close analogy to the six-dimensional case analyzed in [16]. The corresponding inflow is due to (2k)-twisted axions and $C_{(2k)} = C_{(N-2k)}$ when $2k > N/2$. Moreover, as ordinary flat orientifold planes, these couplings involve the Hirzebruch polynomial $\hat{L}(R)$ [20].

Notice that in deriving couplings by factorization there is always the arbitrariness of rescaling each $\chi_{(k)}^{i_k}$ and its dual $\tilde{\chi}_{(k)}^{i_k}$ by two opposite factors. Here and in the following we write the couplings in the most symmetric way. Furthermore, possible couplings that do not give rise to inflow, like couplings only to the axion or only to its corresponding tensor field, are obviously undetectable through this approach.

Recall now that using the condition of non-Abelian anomaly cancellation (3.11), the total anomaly (3.10) simplifies to the expression (3.12) looking like a pure annulus inflow, the Möbius contribution to the mixed $U(1)$-gravitational anomaly being proportional to the corresponding annulus contribution. This observation and the simple form of the anomaly (3.12) strongly suggest that the only effect of the fixed-point couplings should be to rescale the gravitational couplings of the D-branes. One can check that this is indeed what happens. The net effect of the fixed-point couplings is simply to rescale by a factor of 3/2 the gravitational part of those of D-branes. Correspondingly, the total GS term for $N = 1$ sectors, obtained by adding the three couplings (4.3), becomes

$$S_{GS}^{\mathbf{Z}_N} = \sqrt{\frac{2\pi}{N}} \sum_{k \in S} (N_k)^{1/4} \sum_{\alpha=9,5} \alpha_k^\alpha \int \left[\chi_{(k)}^\alpha \wedge Y_{(k)}^\alpha + \tilde{\chi}_{(k)}^\alpha \wedge X_{(k)}^\alpha\right] \ , \tag{4.4}$$

in terms of the 2 and 4-forms

$$X_{(k)}^\alpha = \frac{i\epsilon_k}{2\pi} \mathrm{tr}(\gamma_k F_\alpha) \ ,$$

$$Y_{(k)}^\alpha = -\frac{1}{2(2\pi)^2} \left(\mathrm{tr}(\gamma_k F_\alpha^2) - \frac{1}{32} \mathrm{tr}(\gamma_{k,\alpha})\, \mathrm{tr} R^2\right) \ , \tag{4.5}$$

and the normalized combination of axions

$$\chi_{(k)}^9 = \frac{1}{\sqrt{N_k}} \sum_{i_k=1}^{N_k} \chi_{(k)}^{i_k} \ , \quad \chi_{(k)}^5 = \frac{1}{\sqrt{N_k^3}} \sum_{i_k=1}^{N_k^3} \chi_{(k)}^{i_k} \ . \tag{4.6}$$

Denoting schematically by $\langle\, \chi\tilde{\chi}\, \rangle$ the inflow generated by the presence of these fields in the GS term (4.4), one gets $\langle\, \chi^{\alpha}_{(k)}\, \tilde{\chi}^{\beta}_{(k)}\, \rangle \sim (N_k)^{-1/2}\, |C_k^{\alpha\beta}|$ according to the definitions (4.6), and the inflow induced by (4.4) is given simply by

$$I_{GS}^{\mathbf{Z}_N} = \frac{i}{2N} \sum_{k \in S} \sum_{\alpha,\beta=9,5} \epsilon_k\, C_k^{\alpha\beta}\, X^{\alpha}_{(k)} \wedge Y^{\beta}_{(k)} \;, \tag{4.7}$$

reproducing the contribution of $N=1$ sectors to (3.12).

$\mathbf{Z}_N \times \mathbf{Z}_M$ models

For $\mathbf{Z}_N \times \mathbf{Z}_M$ orientifolds, the situation is similar. States which are untwisted with respect to the $\mathbf{Z}_N$ factor do not contribute to the inflow. However, one has to consider all the twists with respect to the $\mathbf{Z}_M$ factor. In this case, the orientifold projection relates (k,l) and $(N-k, M-l)$ twists, yielding $N'_{k,l}$ linear multiplets from each pair of twisted sectors. The distinct twisted sectors are then labeled by $k = 1, ..., [(N-1)/2]$, $l = 0, 1, ..., M-1$. In general $N'_{k,l} \leq N_{k,l}$ but, as before, it is convenient to keep all the $N_{k,l}$ fields arising in each twisted sector. The $N=1$ sectors come from $(k,l) \in S$, with

$$S = \Big\{ k = 1, ..., [(N-1)/2]\;,\; l = 0, 1, ..., M-1 \,|\, N_{k,l} \neq 0 \Big\} \;.$$

Proceeding as in the $\mathbf{Z}_N$ case, one finds perfectly similar results for the anomalous couplings of D-branes and fixed-points. Again, the contribution of the fixed-points can be rewritten in the same form as the D-brane contributions, by using the condition (3.20). The total GS coupling in the $N=1$ sectors is then found to be:

$$S_{GS}^{\mathbf{Z}_N \times \mathbf{Z}_M} = \sqrt{\frac{2\pi}{NM}} \sum_{(k,l) \in S} (N_{k,l})^{1/4} \sum_{\alpha=9,5} \alpha^{\alpha}_{k,l} \int \Big[\chi^{\alpha}_{(k,l)} \wedge Y^{\alpha}_{(k,l)} + \tilde{\chi}^{\alpha}_{(k,l)} \wedge X^{\alpha}_{(k,l)} \Big] \;, \tag{4.8}$$

where

$$\begin{aligned} X^{\alpha}_{(k,l)} &= \frac{i\epsilon_{k,l}}{2\pi} \mathrm{tr}(\gamma_k \delta_l F_\alpha) \;, \\ Y^{\alpha}_{(k,l)} &= -\frac{1}{2(2\pi)^2} \Big(\mathrm{tr}(\gamma_k \delta_l F_\alpha^2) - \frac{1}{32} \mathrm{tr}(\gamma_{k,\alpha} \delta_{l,\alpha})\, \mathrm{tr} R^2 \Big) \;, \end{aligned} \tag{4.9}$$

and

$$\chi^{9}_{(k,l)} = \frac{1}{\sqrt{N_{k,l}}} \sum_{i_{k,l}=1}^{N_{k,l}} \chi^{i_{k,l}}_{(k,l)} \;,\quad \chi^{5}_{(k,l)} = \frac{1}{\sqrt{N^3_{k,l}}} \sum_{i_{k,l}=1}^{N^3_{k,l}} \chi^{i_{k,l}}_{(k,l)} \;, \tag{4.10}$$

The signs $\epsilon_{k,l}$ and $\alpha^{\alpha}_{k,l}$ are defined similarly to before and one can take the values $\epsilon_k = \mathrm{sign}(s^1_{k,l} s^2_{k,l} s^3_{k,l})$, $\alpha^5_{k,l} = \mathrm{sign}(s^3_{k,l})$, $\alpha^9_{k,l} = \mathrm{sign}(s^1_{k,l} s^2_{k,l} s^3_{k,l})$. Again, one can check that the definitions (4.10) imply that $\langle\, \chi^{\alpha}_{(k,l)}\, \tilde{\chi}^{\beta}_{(k,l)}\, \rangle \sim (N_{k,l})^{-1/2}\, |C^{\alpha\beta}_{k,l}|$, and the inflow induced by the GS couplings (4.8) is given by

$$I_{GS}^{\mathbf{Z}_N \times \mathbf{Z}_M} = \frac{i}{2NM} \sum_{(k,l) \in S} \sum_{\alpha,\beta=9,5} \epsilon_{k,l}\, C^{\alpha\beta}_{k,l}\, X^{\alpha}_{(k,l)} \wedge Y^{\beta}_{(k,l)} \;, \tag{4.11}$$

reproducing the contribution of $N=1$ sectors to (3.21).

4.2 $N=2$ sectors

$N=2$ sectors, i.e. k-twisted sectors containing planes left fixed by the orbifold group, can arise when $kv^i =$ integer for some i, in which case $N^i_k = 0$. A subset of these sectors, the $k = N/2$ sectors of all the even $\mathbf{Z}_N$ models and the $(k = 0, l = 3)$ sector of the $\mathbf{Z}_3 \times \mathbf{Z}_6$ model, do not contribute at all to the inflow, but the field theory explanation for this fact is clear and reported in next section. Among the remaining sectors, only the $k = 2$ sector of the $\mathbf{Z}'_6$ model and the $(k = 1, l = 0, 4)$ sectors of the $\mathbf{Z}_3 \times \mathbf{Z}_6$ model, give a non-vanishing inflow in the 95 sector. The above one is reproduced by adding the following couplings:

$$S'^{\mathbf{Z}'_6}_{GS} = \sqrt{\frac{\pi}{3}}\, 3^{1/4} \sum_{\alpha=9,5} \alpha^\alpha_2 \int \left[\chi^\alpha_{(2)} \wedge Y^\alpha_{(2)} + \tilde{\chi}'^\alpha_{(2)} \wedge X^\alpha_{(2)}\right] \;, \tag{4.12}$$

$$S'^{\mathbf{Z}_3\times\mathbf{Z}_6}_{GS} = \sqrt{\frac{\pi}{9}} \sum_{l=0,4} 3^{1/4} \sum_{\alpha=9,5} \alpha^\alpha_{1,l} \int \left[\chi^\alpha_{(1,l)} \wedge Y^\alpha_{(1,l)} + \tilde{\chi}'^\alpha_{(1,l)} \wedge X^\alpha_{(1,l)}\right] \;. \tag{4.13}$$

The 2 and 4-forms X and Y are the same as before. However, in order to reproduce a vanishing 99/55 inflow and the correct 95 one, two orthogonal combinations of RR fields have to be introduced, χ^α and χ'^α, such that $\langle\chi^{9,5}\tilde{\chi}^{9,5}\rangle = 0$ and $\langle\chi^{9,5}\tilde{\chi}^{5,9}\rangle = 1$. The crucial difference with the previously analyzed $N = 1$ sectors is that $\tilde{\chi}'^{9,5}$ is clearly not the dual of $\chi^{9,5}$.

The massless closed string content in these twisted sectors with non-vanishing inflow is always the same [10, 21]. There are six linear multiplets, but only three of these arise inside the world-volume of the D5-branes, the other three being combinations of fields living outside the world-volume of the D5-branes.

At this point, one has to make some assumption in order to extract a definite answer for the combination of axions χ^α and χ'^α. A first reasonable assumption is that D5-branes couple only to the first three linear multiplets arising within their world-volume, whereas D9-branes can couple to all six of them. Another equally reasonable but less obvious assumption is to require a symmetry between all similar fixed-points, as done for the $N = 1$ sectors. Unfortunately, these two assumptions are together incompatible with the inflows computed before, and one of them has necessarily to be relaxed. In this way, however, even making use of T-duality, which relates the D5 and D9-brane couplings to each other, one is left with some free parameters which cannot be completely fixed.

We would like to stress that the results of section three nevertheless demonstrate anomaly cancellation, and that a net inflow takes place also in certain $N = 2$ sectors. However, we conclude that the factorization approach followed here is not powerful enough to fix unambiguously the combinations of fields entering the corresponding anomalous couplings. It should be possible to obtain the precise coefficient through a more direct computation on the disk and the crosscap.

4.3 $N = 4$ sectors

A comment is in order also about the untwisted $N = 4$ sector. As shown in section three, the inflow in this sector is trivially vanishing. However, there are definitely WZ couplings, although they do not interfere to give an inflow. This can be understood by looking at this sector from the $D = 10$ Type I point of view and compactify the usual anomalous couplings of the D9-branes and the O9-plane on T^6, and those of the D5-branes and O5-planes (whenever these occur) on a T^2. Since the curvatures are non-trivial only in four-dimensional non-compact space, the only terms surviving this reduction are those obtained by integrating the ten-dimensional 6-form on T^6, yielding a four-dimensional scalar, and the six-dimensional 2-form on T^2, yielding another four-dimensional scalar. Since no couplings to the two-forms dual to these scalars appear, no inflow of anomaly is induced.

5. Field theory analysis

One of the consequences of the presence of the GS terms (4.4), (4.8), (4.12) and (4.13) in the low-energy effective action is the spontaneous breakdown of various combinations of $U(1)$ factors [9, 22, 8], involved in the would-be $U(1)$-gauge/gravitational anomalies. The Higgs mechanism is due to the couplings to the 2-forms $\tilde{\chi}^\alpha$ or $\tilde{\chi}'^\alpha$, which modify (after dualization) the kinetic terms for the corresponding axions χ^α or χ'^α by a shift involving the $U(1)$ gauge fields. Gauge invariance requires then that these fields transform inhomogeneously under the appropriate gauge transformation, and the scalar is eaten by the gauge field. The combinations of $U(1)$ gauge fields involved in this mechanism are reported in the appendix for each model. As usual, care is needed in interpreting string theory results in terms of couplings in a low-energy effective action. In our case, particular attention is needed because of the well-known chiral multiplet - linear multiplet duality [23]. We do not discuss this issue here, since it has has been extensively analyzed in [14, 10] in the context of D=4 $N = 1$ IIB orientifolds, and recall simply that the string theory results derived in last section can be interpreted most directly in the linear multiplet formulation.

The models considered here have $N = 1$ supersymmetry. This allows to fix the tree-level form of two other CP-even couplings, related by supersymmetry to the GS couplings above: Fayet-Iliopoulos (FI) terms [5] and gauge couplings (GC) depending on the NSNS scalars m^α, partners of the twisted RR fields. As well known, FI terms can trigger both supersymmetry and gauge symmetry breaking, and a supersymmetric vacuum with unbroken non-Abelian gauge group requires a vanishing FI term. This is particularly important since the vacuum expectation values (vev) of the scalars m^α is fixed by the FI terms.

[5] In this case, only a tree-level contribution seems to be present [24].

Consider first the $N = 1$ sectors. The FI terms can be read directly from the $\tilde{\chi} \wedge X$ couplings in the GS terms of last section, and from the $\chi \wedge Y$ couplings one gets the following GC corrections:

$$S_{GC}^{\mathbf{Z}_N} = \sqrt{\frac{2\pi}{N}} \sum_{k \in S} (N_k)^{1/4} \sum_{\alpha=9,5} \alpha_k^\alpha \int d^4x \, m_{(k)}^\alpha \, \mathrm{tr}(\gamma_k F_{\alpha,\mu\nu} F_\alpha^{\mu\nu}) \,, \tag{5.1}$$

$$S_{GC}^{\mathbf{Z}_N \times \mathbf{Z}_M} = \sqrt{\frac{2\pi}{NM}} \sum_{(k,l) \in S} (N_{k,l})^{1/4} \sum_{\alpha=9,5} \alpha_{k,l}^\alpha \int d^4x \, m_{(k,l)}^\alpha \, \mathrm{tr}(\gamma_k \delta_l F_{\alpha,\mu\nu} F_\alpha^{\mu\nu}) \,. \tag{5.2}$$

Importantly, the FI terms and GC involve the same combination of scalars. For unbroken supersymmetry and non-abelian symmetry, a vanishing FI term implies that the vev of the combination of scalars appearing in (5.1) and (5.2) vanish as well. Unless one breaks the gauge group and/or supersymmetry, there are no gauge coupling corrections. Apart from an overall coefficient, our results (5.1)-(5.2) reproduce those of [14], extending them to other models and gauge fields in the 55 sector.

As expected, $N = 2$ sectors are more subtle. For Z_2-twisted $N = 2$ sectors, which have fixed-planes along the world-volume of the D5-branes, the inflow vanishes. This fact alone is not sufficient to rule out corrections to gauge couplings, but supersymmetry implies that couplings like (5.1)-(5.2) are nonetheless forbidden. Indeed, for these sectors the compact space is effectively $R^4 \times T^2 \times T^4/G$, with $N = 2$ supersymmetry and the D5-branes wrapped on T^2, and one can therefore treat them as $\mathbf{Z}_2$-twisted sectors of dimensionally reduced $D = 6$ $N = 1$ orientifolds. In these sectors the only RR states exchanged are scalars, belonging to $D = 6$ hypermultiplets [16]. Since these cannot couple to gauge kinetic terms in $D = 6$ [25], these couplings are forbidden also in the $D = 4$ $N = 1$ model. The situation is different for $N = 2$ sectors with fixed-planes outside the D5-branes world-volume. In these cases, the inflow is either vanishing ($k = 3$ in $\mathbf{Z}_{12}$ and $(k = 0, l \neq 0, 3)$ in $\mathbf{Z}_3 \times \mathbf{Z}_6$) or restricted to the 95 sector ($k = 2$ in $\mathbf{Z}_6'$ and $(k = 1, l = 0, 4)$ in $\mathbf{Z}_3 \times \mathbf{Z}_6$). In the first case, a dependence of the gauge couplings on the corresponding scalars is allowed if no FI term is generated at all. In the second case, the combination of scalars appearing in the gauge couplings and the FI terms have to be orthogonal, and again a gauge coupling dependence is allowed. Summarizing, in the orbifold limit where all these scalars have zero vev, no gauge coupling correction is present in any case, but there is the interesting possibility of orbifold deformations leading to a non-vanishing tree-level correction to the gauge couplings, even for unbroken gauge symmetry. A further study is definitely needed to get a better understanding of these sectors.

Analogously to the $F \wedge F$ terms, supersymmetry, or better supergravity, also fixes the form of the various CP-even terms related to the $R \wedge R$ couplings. Being higher order in derivatives, we will not discuss the explicit form of these terms.

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A. Appendix

We report here the explicit combination of the various would-be anomalous $U(1)$ factors that become massive for each model. In order to find their right combination, it is important to write the GS couplings of section four in terms of physical fields only. All the combinations are symmetric in the $9,5$ exchange, as expected by T-duality.

$\mathbf{Z}_3$

All the fields appearing in (4.4) are physical and the unique $U(1)$ present in the model gets a mass.

$\mathbf{Z}_7$

Again, all the 7 fields appearing in each of the three sectors in (4.4) are physical. An independent combination of $U(1)$ factors is present in each sector. Correspondingly, all of the three $U(1)$'s present in the model get a mass.

$\mathbf{Z}_6$

The 3 fields in the $k = 1$ sector are all physical. In the $k = 2$ sector, $\chi_{(2)}^{1,2,3}$ are again all physical, whereas the other 24 fields give rise to only 12 propagating combinations given by $1/\sqrt{2}(\chi^i_{(2)} + \chi^{i+12}_{(2)})$, $i = 4, ..., 15$. The three massive $U(1)$ fields are

$$
\begin{aligned}
A_{(1)} &= \mathrm{tr}(\gamma_1 A_9) + \mathrm{tr}(\gamma_1 A_5) \ , \\
A_{(2,3)} &= \mathrm{tr}(\gamma_2 A_{9,5}) + \frac{1}{3}\mathrm{tr}(\gamma_2 A_{5,9}) \ .
\end{aligned}
$$

$\mathbf{Z}_6'$

All the fields are physical. The four massive $U(1)$'s are

$$
\begin{aligned}
A_{(1,2)} &= \mathrm{tr}(\gamma_1 A_{5,9}) - \frac{1}{2}\mathrm{tr}(\gamma_1 A_{9,5}) \ , \\
A_{(3,4)} &= \mathrm{tr}(\gamma_2 A_{9,5}) + c\,\mathrm{tr}(\gamma_2 A_{5,9}) \ ,
\end{aligned}
$$

where c is an arbitrary coefficient. The form of the gauge fields $A_{(3,4)}$ is dictated by T-duality only, since they get a mass by higgsing axions in $N = 2$ sectors, where our factorization procedure is ambigous.

$\mathbf{Z}_{12}$

Fields are identified under a $\mathbf{Z}_N$ projection only in the $k=4$ sector. In this case, $\chi_{(4)}^{1,2,3}$ are physical, whereas the other 24 fields give rise to 6 propagating combinations given by $1/2(\chi_{(4)}^i + \chi_{(4)}^{i+6} + \chi_{(4)}^{i+12} + \chi_{(4)}^{i+18})$, i=4,...,9. The five massive gauge fields are

$$\begin{aligned} A_{(1,2)} &= \mathrm{tr}(\gamma_{1,2} A_9) - \mathrm{tr}(\gamma_{1,2} A_5) \ , \\ A_{(3,4)} &= \mathrm{tr}(\gamma_4 A_{9,5}) + \frac{1}{3}\mathrm{tr}(\gamma_4 A_{5,9}) \ , \\ A_{(5)} &= \mathrm{tr}(\gamma_5 A_9) + \mathrm{tr}(\gamma_5 A_5) \ . \end{aligned}$$

$\mathbf{Z}_3 \times \mathbf{Z}_3$

All the 27 fields are physical and a single $U(1)$ gets a mass:

$$A = \mathrm{tr}(\gamma_1 \delta_2 A_9) \ .$$

$\mathbf{Z}_3 \times \mathbf{Z}_6$

Fields are identified in the $(k,l) = (1,1),(1,2),(1,3)$ sectors. In all these sectors, the first three states, *i.e.* those coupling also to D5-branes, are physical. The remaining physical combinations are $1/\sqrt{3}(\chi_{(1,1)}^i + \chi_{(1,1)}^{i+3} + \chi_{(1,1)}^{i+6})$, $i=4,5,6$, $1/\sqrt{2}(\chi_{(1,2)}^i + \chi_{(1,2)}^{i+12})$, $i=4,...,12$, and $1/\sqrt{3}(\chi_{(1,3)}^i + \chi_{(1,3)}^{i+3} + \chi_{(1,3)}^{i+6})$, $i=4,5,6$. The eleven massive $U(1)$'s are

$$\begin{aligned} A_{(1,2)} &= \mathrm{tr}(\gamma_1 A_{9,5}) + d\,\mathrm{tr}(\gamma_1 A_{5,9}) \ , \\ A_{(3,4)} &= \mathrm{tr}(\gamma_1 \delta_1 A_{9,5}) - \frac{1}{2}\mathrm{tr}(\gamma_1 \delta_1 A_{5,9}) \ , \\ A_{(5,6)} &= \mathrm{tr}(\gamma_1 \delta_2 A_{9,5}) - \frac{1}{3}\mathrm{tr}(\gamma_1 \delta_2 A_{5,9}) \ , \\ A_{(7,8)} &= \mathrm{tr}(\gamma_1 \delta_3 A_{9,5}) - \frac{1}{2}\mathrm{tr}(\gamma_1 \delta_3 A_{5,9}) \ , \\ A_{(9,10)} &= \mathrm{tr}(\gamma_1 \delta_4 A_{9,5}) + e\,\mathrm{tr}(\gamma_1 \delta_4 A_{9,5}) \ , \\ A_{(11)} &= \mathrm{tr}(\gamma_1 \delta_5 A_9) + \mathrm{tr}(\gamma_1 \delta_5 A_5) \ . \end{aligned}$$

Again, the arbitrary coefficients d and e reflect the ambiguity in the factorization procedure for $N=2$ sectors. For this reason, we have not explicitly checked whether the gauge fields above are always independent or not.

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